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# An algorithm for multiobjective integer nonlinear fractional programming problem under fuzziness

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Abstract. In this paper, a solution algorithm to fuzzy multiobjective integer nonlinear fractional programming problem (FMOINLFP) is suggested. The problem of concern involves fuzzy parameters in the objective functions. In order to defuzzify the problem, the concept of $\alpha$-level set of the fuzzy number is given and for obtaining an efficient solution to the problem (FMOINLFP), a linearization technique is presented to develop the solution algorithm. In addition, an illustrative example is included to demonstrate the correctness of the proposed solution algorithm.

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## 1. Introduction

Fuzzy integer linear and nonlinear fractional programming problems with multiple objective is an important field of research and has not received attention as much as did to fuzzy multiple objective linear and nonlinear fractional programming problems.

Integer linear fractional programming problem with multiple objectives (MOILFP) is an important field of research and has not received as much attention as did multiple objective linear fractional programming. In [3], an exact method for discrete multiobjective linear fractional optimization has been developed using a branch and cut algorithm to generate the whole integer efficient solutions of the MOILFP problem.

Literature survey reveals wide applications of fractional programming in different areas ranging from engineering to economics. For comprehensive review of the work in this filed, we refer to [13].

In our previous paper [12], we have presented an algorithm to solve multiobjective integer linear fractional programming problem (FMOILFP) with fuzzy coefficients in the right-hand side of the constraint functions. The basic idea of the computational phase of the suggested algorithm in [12] was based mainly upon a modification of Isbell-Marlow method together with the branch and bound technique.

In this paper, an attempt is made to study multiobjective integer nonlinear fractional programming problem (FMOINLFP) with fuzzy coefficients in the objective functions. The problem formulation is introduced in Section 2. Fuzzy notations and definitions used throughout this paper are presented in Section 3. In Section4, a linearization technique is described. An algorithm to solve problem (FMOINLFP) is developed in Section 5. An illustrative example is given in Section 6 to clarify the solution algorithm. Section 7 provides some concluding remarks.

## 2. Problem Formulation

The purpose of this paper is to develop a solution algorithm for solving the following multiobjective integer non-linear fractional programming problem involving fuzzy parameters in the objective functions [FMOINLFP]:

$$\text{(FMOINLFP)}: \begin{cases} \max z_1(x) = \frac{c_1^T x + \widetilde{\theta}_1^T x + \alpha_1}{d_1^T x + \beta_1}, \\ \max z_2(x) = \frac{c_2^T x + \widetilde{\theta}_2^T x + \alpha_2}{d_2^T x + \beta_2}, \\ \qquad\vdots \\ \max z_k(x) = \frac{c_k^T x + \widetilde{\theta}_k^T x + \alpha_k}{d_k^T x + \beta_k}, \end{cases} \tag{2.1}$$

Subject to

$$x \in M.$$

In problem (2.1), $c, d \in R^n$ for each objective $l$, $l = [1, 2, ..., k]$ and $\alpha_i \beta_i \in R$. The set $M$ is defined as the feasible region and might be, for example, of the form

$$M = \{x \in R^n \mid Ax \leq b, x \geq 0 \text{ and integer}\} \tag{2.2}$$

where $A$ is an $m \times n$ real matrix, $x$ is an $n$-vector of integer decision variables, $b$ is an $m$-vector of the constraints right-hand sides, $R^n$ is the $n$-dimensional Euclidean space and $T$ denotes the transpose.

It is assumed that $\widetilde{\theta_{li}}$ is an $k \times n$ real matrix of fuzzy parameters and for simplicity let

$$\widetilde{\theta} = \begin{pmatrix} \widetilde{\theta}_{11} & \widetilde{\theta}_{12} & \dots & \widetilde{\theta}_{1n} \\ \widetilde{\theta}_{21} & \widetilde{\theta}_{22} & \dots & \widetilde{\theta}_{2n} \\ \dots & \dots & \dots & \dots \\ \widetilde{\theta}_{k1} & \widetilde{\theta}_{k2} & \dots & \widetilde{\theta}_{kn} \end{pmatrix}.$$

Moreover, $M$ is a compact set and that $d_{li}^T x + \beta_l > 0$ for all $x \in M$ is nonconvex polyhedron in general.

The set of constraints $Ax \leq b$, $x \geq 0$ will be denoted throughout this paper by $M_R$ and can be obtained by dropping the integer requirement on the decision variables $x_J$ for all $J = 1, 2, ..., n$ in (2.2) above.

In what follows, an equivalent fuzzy multiobjective nonlinear fractional programming problem associated with problem (2.1) can be stated with the help of the cutting-plane technique [6, 8] and may be written in the form:

$$\text{(FMOINLFP)}: \begin{cases} \max z_1(x) = \frac{c_1^T x + \widetilde{\theta}_1^T x + \alpha_1}{d_1^T x + \beta_1}, \\ \max z_2(x) = \frac{c_2^T x + \widetilde{\theta}_2^T x + \alpha_2}{d_2^T x + \beta_2}, \\ \qquad\qquad \vdots \\ \max z_k(x) = \frac{c_k^T x + \widetilde{\theta}_k^T x + \alpha_k}{d_k^T x + \beta_k}, \end{cases} \tag{2.3}$$

Subject to

$$x \in [M]$$

where $[M]$ is defined as the convex hull of the set of feasible solutions $M$ defined by (2.2) and the point to be noted here is that the efficient solution of problem (2.1) is the same efficient solution of problem (2.3), (see [10]).

In what follows, we consider the equivalent fuzzy multiobjective nonlinear fractional problem (2.3) in the following form:

$$\text{(FMOINLFP)}: \begin{cases} \max z_1(x) = \frac{c_1^T x + \widetilde{\theta}_1^T x + \alpha_1}{d_1^T x + \beta_1}, \\ \max z_2(x) = \frac{c_2^T x + \widetilde{\theta}_2^T x + \alpha_2}{d_2^T x + \beta_2}, \\ \qquad\qquad \vdots \\ \max z_k(x) = \frac{c_k^T x + \widetilde{\theta}_k^T x + \alpha_k}{d_k^T x + \beta_k}, \end{cases} \tag{2.4}$$

Subject to

$$x \in M_R^{(s)}$$

where $M_R^{(s)}$ is defined as:

$$M_R^{(s)} = \{x \in R^n \mid A^{(s)} x \leq b^{(s)},\ x \geq 0\} \tag{2.5}$$

In addition,

$$A^{(s)} = \begin{bmatrix} A \\ \vdots \\ a_1 \\ \vdots \\ a_s \end{bmatrix} \quad \text{and} \quad b^{(s)} = \begin{bmatrix} b \\ \vdots \\ b_1 \\ \vdots \\ b_s \end{bmatrix} \tag{2.6}$$

are the original constraint matrix $A$ and the right-hand side vector $b$, respectively, with $s$-additional constraints, each corresponding to an efficient non-redundant cut in the form $a_i x \leq b_i$, where $M_R^{(s)} = [M]$ and for more details, the reader is referred to [11].

Now, using the nonnegative weighted sum method [2], then problem (2.4) will take the following form with a single-objective function:

$$\text{(FMOINLFP)} \; :$$

$$\max\left\{w_1\left(\frac{c_1^T x+\widetilde{\theta}_1^T x+\alpha_1}{d_1^T x+\beta_1}\right)+w_2\left(\frac{c_2^T x+\widetilde{\theta}_2^T x+\alpha_2}{d_2^T x+\beta_2}\right)+\cdots +w_k\left(\frac{c_k^T x+\widetilde{\theta}_k^T x+\alpha_k}{d_k^T x+\beta_k}\right)\right\} \tag{2.7}$$

Subject to

$$x \in [M].$$

## 3. Fuzzy Concepts and Notations

The fuzzy number is defined differently by many authors. The most frequently used definition belongs to a trapezoidal fuzzy type as follows:

**Definition 3.1** ([5])**.** It is appropriate to recall that a real fuzzy number $\widetilde{P}$, is a continuous fuzzy subset from the real line $R$ whose membership function $\mu_{\widetilde{P}}(P)$ is defined by:

1. A continuous mapping from $R$ to the closed interval $[0, 1]$,
2. $\mu_{\widetilde{P}}(P) = 0$ for all $P \in (-\infty, P_1]$,
3. $\mu_{\widetilde{P}}(P)$ is strictly increasing on $[P_1, P_2]$,
4. $\mu_{\widetilde{P}}(P) = 1$ for all $P \in [P_2, P_3]$,
5. $\mu_{\widetilde{P}}(P)$ is strictly decreasing on $[P_3, P_4]$,
6. $\mu_{\widetilde{P}}(P) = 0$ for all $[P_4, +\infty)$.

Figure 1. illustrates the graph of a possible shape of a membership function of a fuzzy number $\widetilde{P}$.

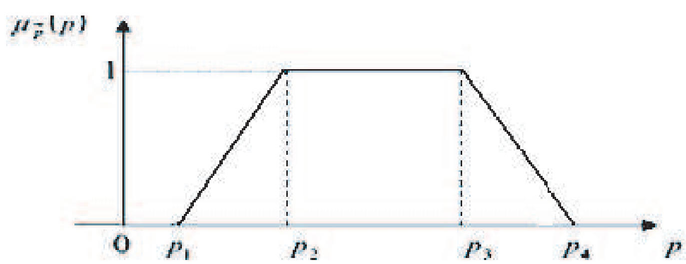


Figure 1. Membership function of a fuzzy number $\widetilde{P}$

Here, the matrix of fuzzy parameters $\widetilde{\theta}$ involved in problem (FMOINLFP) is a matrix of fuzzy numbers whose membership function is denoted by $\mu_{\widetilde{\theta}}(\theta)$.

In the following we give the definition of the $\alpha$-level set or $\alpha$-cut of the fuzzy matrix $\widetilde{\theta}$.

**Definition 3.2** ([5])**.** The $\alpha$-level set of the matrix of fuzzy parameters $\widetilde{\theta}$ in the problem (FMOINLFP) is defined as the ordinary set $L_\alpha(\widetilde{\theta})$ for which the degree of its membership function exceeds the level $\alpha \in [0,1]$, where

$$L_\alpha(\widetilde{\theta}) = \{\theta \in R^n \mid \mu_{\widetilde{\theta}}(\theta) \geq \alpha\} \tag{3.1}$$

For a certain degree $\alpha = \alpha^* = [0,1]$, estimated by the decision maker. The problem (FMOINLFP) (2.7) can be understood as the following nonfuzzy $\alpha$-multiobjective integer nonlinear fractional programming problem ($\alpha$-MOINLFP) :

$$\begin{aligned} &(\alpha\text{-MOINLFP}):\\ &\max\left\{w_1\left(\tfrac{c_1^T x+\widetilde{\theta}_1^T x+\alpha_1}{d_1^T x+\beta_1}\right)+w_2\left(\tfrac{c_2^T x+\widetilde{\theta}_2^T x+\alpha_2}{d_2^T x+\beta_2}\right)+\dots\right.\\ &\qquad\qquad\left.+w_k\left(\tfrac{c_k^T x+\widetilde{\theta}_k^T x+\alpha_k}{d_k^T x+\beta_k}\right)\right\} \end{aligned} \tag{3.2}$$

Subject to

$$x \in M(\theta),$$

where

$$M(\theta) = \{x \in R^n \mid A^{(s)}x \leq b^{(s)},\ x \geq 0,\ \theta \in L_\alpha(\widetilde{\theta})\}.$$

If should be emphasized here in the ($\alpha$-MOINLFP) (3.2) above that the matrix of parameters $\theta$ is treated as a matrix of decision variables rather than constants. Depending on the basic definition of the $\alpha$-level set of the fuzzy numbers, we introduce the concept of the $\alpha$-efficient solution to the ($\alpha$-MOINLFP) (3.2) in the following definition.

**Definition 3.3** ([5])**.** A point $x^* \in M(\theta^*)$ is said to be an $\alpha$-efficient solution to problem($\alpha$-MOINLFP), if and only if there does not exist another $x \in M(\theta)$, $\theta \in L_\alpha(\widetilde{\theta})$ such that $z_l(x) \geq z_l(x^*)$, $(l = 1, 2, ..., k)$ with strict inequality holding for at least one $l$, where the corresponding values of parameters $\theta^*$ are called $\alpha$-level optimal parameters.

Throughout this paper, a membership function of the fuzzy matrix $\widetilde{\theta}$ in the following form will be elicited:

$$\mu_{\widetilde{P}}(P) = \begin{cases} 0, & \theta \leq P_1,\\ 1-\left(\frac{P-P_2}{P_1-P_2}\right)^2, & P_1 \leq \theta \leq P_2,\\ 1, & P_2 \leq \theta \leq P_3,\\ 1-\left(\frac{P-P_3}{P_4-P_3}\right)^2, & P_3 \leq \theta \leq P_4,\\ 0, & \text{otherwiae} \end{cases} \tag{3.3}$$

Now, before we go any further, problem ($\alpha$-MOINLFP) (3.2) can be rewritten as follows:

$$\begin{aligned} &(\alpha\text{-MOINLFP}):\\ &\max\left\{w_1\left(\tfrac{c_1^T x+\widetilde{\theta}_1^T x+\alpha_1}{d_1^T x+\beta_1}\right)+w_2\left(\tfrac{c_2^T x+\widetilde{\theta}_2^T x+\alpha_2}{d_2^T x+\beta_2}\right)+\dots\right.\\ &\qquad\qquad\left.+w_k\left(\tfrac{c_k^T x+\widetilde{\theta}_k^T x+\alpha_k}{d_k^T x+\beta_k}\right)\right\} \end{aligned} \tag{3.4}$$

Subject to

$$\begin{aligned} x \in M(\theta) = &\{x \in R^n \mid A^{(s)}x \leq b^{(s)},\ l_{li}^{(0)} \leq \theta_{li} \leq L_{li}^{(0)},\\ &(l = 1, 2, \dots, k),\ (i = 1, 2, \dots, n),\ x \geq 0\} \end{aligned}$$

Note that the constraint $\theta \in L_\alpha(\widetilde{\theta})$ in problem ($\alpha$-MOINLFP) (3.4) stated above has been replaced by the equivalent one $l_{li}^0 \leq \theta_{li} \leq L_{li}^0$ where $l_{li}^0$ and $L_{li}^0$ are the lower and the upper bounds on the variables $\theta_{li}$.

In what follows, a linearization procedure is suggested to deal with problem (3.4). This procedure depends basically on the linearity nature of the trapezoidal membership function defined by (3.1) . On the other hand, we should report that a different membership function in the non-linear case will require a modification in the linearization process and this point is recommended to be handled as a future research work.

## 4. Linearization Procedure (see [1])

The nonlinearity in the objective's numerators of problem (2.6) can be treated using the following transformation:

$$\gamma_l^T = x_i\theta_{li}^T \text{ for each objective } l,\ l = \{1, 2, \dots, k\},\ i = \{1, 2, \dots, n\} \tag{4.1}$$

Consequently problem (2.6) becomes:

$$\max\left\{w_1\left(\tfrac{c_1^T x+\gamma_1^T x+\alpha_1}{d_1^T x+\beta_1}\right) + w_2\left(\tfrac{c_2^T x+\gamma_2^T x+\alpha_2}{d_2^T x+\beta_2}\right) + \cdots + w_k\left(\tfrac{c_k^T x+\gamma_k^T x+\alpha_k}{d_k^T x+\beta_k}\right)\right\} \tag{4.2}$$

Subject to

$$\begin{aligned} x \in M(\theta) = &\{x \in R^n \mid A^{(s)}x \leq b^{(s)}, x_i l_{li}^{T(0)} \leq \gamma_l^T \leq x_i L_{li}^{T(0)},\\ &(l = 1, 2, \dots, k),\ (i \in J \subseteq (1, 2, \dots, n),\ x \geq 0\} \end{aligned}$$

Using the parametric approach of Dinkelbaeh [4] and Jagannathan [7] for the scalar fractional programming problem, we consider the following optimization problem:

$$\max\left\{\begin{array}{l} w_1\left((c_1^T x + \gamma_1^T x + \alpha_1) - v_1(d_1^T x + \beta_1)\right) + \\ w_2\left((c_2^T x + \gamma_2^T x + \alpha_2) - v_2(d_2^T x + \beta_2)\right) + \cdots \\ \cdots + w_k\left((c_k^T x + \gamma_k^T x + \alpha_k) - v_k(d_k^T x + \beta_k)\right) \end{array}\right\} \tag{4.3}$$

Subject to

$$\begin{aligned} x \in M(\gamma) = &\{x \in R^n \mid A^{(s)}x \leq b^{(s)}, x_i l_{li}^{T(0)} \leq \gamma_l^T \leq x_i L_{li}^{T(0)},\\ &(l = 1, 2, \dots, k),\ (i \in J \subseteq (1, 2, \dots, n),\ x \geq 0\} \end{aligned}$$

where $\sum_{l=1}^k = 1$ and $v_l = z_l(x_1^0, x_2^0, \dots, x_n^0)$.

**Theorem 4.1** ([1])**.** *The solution of the problem (4.3) can be obtained by solving $2^r$ problems, $r \leq \acute{n}$ where $r$ is the number of decision variables from the set $J^c$ for which one of their coefficient whether in the objective functions or in the constraints is fuzzy, where $\acute{n}$ is the cardinality of $J^c$.*

## 5. Solution algorithm

In this section, a solution algorithm to solve fuzzy multiobjective non-linear fractional programming problem (FMOINLFP) is described in a series of steps. The suggested algorithm can be summarized in the following manner:

Step 0. Characterize the set $[M]_R^{(s)} = [M]$ (See [10, 11]),

Step 1. Use the weighted sum method [2] to convert the fuzzy non-linear multi-objective fractional programming problem (2.4) to single-objective problem (2.7)

Step 2. Start with an initial $\alpha$-level set degree $\alpha = \alpha^* = 0$

Step 3. Choose

$$\widetilde{\theta} = \begin{pmatrix} \widetilde{\theta}_{11} & \widetilde{\theta}_{12} & \dots & \widetilde{\theta}_{1n} \\ \widetilde{\theta}_{21} & \widetilde{\theta}_{22} & \dots & \widetilde{\theta}_{2n} \\ \dots & \dots & \dots & \dots \\ \widetilde{\theta}_{k1} & \widetilde{\theta}_{k2} & \dots & \widetilde{\theta}_{kn} \end{pmatrix},$$

the matrix of the fuzzy parameters $\widetilde{\theta}$ in problem (FMOINLFP) (2.7), to elicit a membership function satisfying assumptions (1)-(6) in Definition 1 in the form of $\mu_{\widetilde{\theta}}(\theta)$.

Step 4. Convert problem (FMOINLFP) (2.7) into its nonfuzzy version ($\alpha$-MOINLFP) (3.4).

Step 5. Linearization Procedure:

(a) Let $\gamma_l^T = x_i \theta_{li}^T$ for each objective $l$, $l = \{1, 2, \dots, k\}$, $i = \{1, 2, \dots, n\}$ in the objectiveí≫s numerators of problem (3.4)

(b) Rewrite the problem ( $\alpha$-MOINLFP) in the form of problem (4.2)

(c) Convert problem ($\alpha$-MOINLFP) (4.2) by using the parametric approach of Dinkelbaeh [4] and Jagannathan [7] for the scalar fractional programming in the form of problem (4.3)

(d) Solve the $2^r$ different problems by using LINGO [9] software to obtain $2^r$ of $\alpha$-efficient solution to choose the required one.

Step 6. Set $\alpha = (\alpha^* + Step) \in [0, 1]$ and go to Step 2.

Step 7. Repeat the above procedure until the interval $[0, 1]$ is fully exhausted. Then, stop.

## 6. An Illustrative Example

In this section, an illustrative example is given to clarify the proposed solution algorithm. This example is adapted from one appearing in Chergui and Moulaü [3] and the LINGO [9] software package is used in the computational process.

The problem to be solved here is the following multiobjective integer nonlinear fractional problem involving fuzzy vector of parameters $\widetilde{\theta}$ in the objective functions:

$$\text{(FMOINLFP)}: \begin{cases} \max z_1(x) = \frac{(1+2\widetilde{\theta}_1)x_1 - 4}{-x_2+3}, \\ \max z_2(x) = \frac{-(2+\widetilde{\theta}_2)x_1 + 4}{x_2+1}, \\ \max z_3(x) = -(1 + \widetilde{\theta}_3)x_1 + x_2, \end{cases}$$

Subject to

$$\begin{aligned} -x_1 + 4x_2 &\leq 0 \\ 2x_1 - x_2 &\leq 8 \\ x_1, x_2 &\geq 0, \text{ and integers.} \end{aligned}$$

The convex hull $M_R^{(s)}$ is given by:

$$[M] = \{x \in R^2 \mid -x_1 + 4x_2 \leq 0, 2x_1 - x_2 \leq 8, x_1 \leq 4, x_1, x_2 \geq 0\}$$

where $S = 1$ an efficient Gomory cut: $x_1 \leq 4$ and then problem (FMOINLFP) can be formulated as:

$$(\text{FMOINLFP}) : \begin{cases} \max z_1(x) = \frac{x_1 + 2\widetilde{\theta}_1 x_1 - 4}{-x_2 + 3}, \\ \max z_2(x) = \frac{-2x_1 + \widetilde{\theta}_2 x_1 + 4}{x_2 + 1}, \\ \max z_k(x) = -(x_1 - \widetilde{\theta}_3)x_1 + x_2, \end{cases}$$

Subject to

$$x \in [M]$$

Using the weighting method [2], the multiobjective nonlinear fractional programming problem can be converted to a single-objective nonlinear fractional programming problem as:

$$p_1(w) : \max = w_1 z_1 + w_2 z_2 + w_3 z_3$$

Subject to
where $\sum_{i=1}^{3} w_i = 1$. Therefore, the above problem will take the following form:

$$\begin{aligned} p_1(w) : \max = \Big\{ \Big( \tfrac{1}{4} \Big[ \tfrac{x_1 + 2\widetilde{\theta}_1 x_1 - 4}{-x_2 + 3} \Big] \Big) + \Big( \tfrac{1}{2} \Big[ \tfrac{-2x_1 + \widetilde{\theta}_2 x_1 + 4}{x_2 + 1} \Big] \Big) \\ + \Big( \tfrac{1}{4} \Big[ -x_1 - \widetilde{\theta}_3 x_1 + x_2 \Big] \Big) \Big\} \end{aligned}$$

Subject to

$$\begin{aligned} -x_1 + 4x_2 &\leq 0 \\ 2x_1 - x_2 &\leq 8 \\ x_1 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

where $w_1 = \frac{1}{4} = w_3$, $w_2 = \frac{1}{2}$.

By using the membership function to convert the fuzzy problem $p_1(w)$ to non-fuzzy. Let $\alpha = 0.36$. The membership function corresponding to the fuzzy numbers $\widetilde{\theta}_1, \widetilde{\theta}_2, \widetilde{\theta}_3$ are given by

$$\mu_{\widetilde{\theta}_1}(\theta_1) = \begin{cases} 0, & \theta_1 \leq P_1, \\ 1 - \left( \frac{\theta_1 - P_2}{P_1 - P_2} \right)^2, & P_1 \leq \theta_1 \leq P_2, \\ 1, & P_2 \leq \theta_1 \leq P_3, \\ 1 - \left( \frac{\theta_1 - P_3}{P_4 - P_3} \right)^2, & P_3 \leq \theta_1 \leq P_4, \\ 0, & \text{otherwiae} \end{cases}$$

$$\mu_{\widetilde{\theta}_2}(\theta_2) = \begin{cases} 0, & \theta_2 \le P_1, \\ 1 - \left(\frac{\theta_2 - P_2}{P_1 - P_2}\right)^2, & P_1 \le \theta_2 \le P_2, \\ 1, & P_2 \le \widetilde{\theta}_1 \le P_3, \\ 1 - \left(\frac{\theta_2 - P_3}{P_4 - P_3}\right)^2, & P_3 \le \theta_2 \le P_4, \\ 0, & \text{otherwiae} \end{cases}$$

$$\mu_{\widetilde{\theta}_3}(\theta_3) = \begin{cases} 0, & \theta_3 \le P_1, \\ 1 - \left(\frac{\theta_3 - P_2}{P_1 - P_2}\right)^2, & P_1 \le \theta_3 \le P_2, \\ 1, & P_2 \le \theta_3 \le P_3, \\ 1 - \left(\frac{\theta_3 - P_3}{P_4 - P_3}\right)^2, & P_3 \le \theta_3 \le P_4, \\ 0. & \text{otherwiae} \end{cases}$$

Let also the fuzzy parameters $\widetilde{\theta}_1, \widetilde{\theta}_2, \widetilde{\theta}_3$ are given by the following fuzzy numbers listed in the table below:

| | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|
| $\theta_1$ | 0 | 0.25 | 0.75 | 1 |
| $\theta_2$ | 0 | 0.5 | 0.75 | 1.25 |
| $\theta_3$ | 0 | 0.3 | 0.6 | 0.9 |

It is easy to get:

$$0.05 \le \theta_1 \le 0.95$$
$$0.1 \le \theta_2 \le 1.15$$
$$0.06 \le \theta_3 \le 0.84$$

The problem will be

$$p_2(w) : \max = \left\{ \left( \frac{1}{4} \left[ \frac{x_1 + 2\widetilde{\theta}_1 x_1 - 4}{-x_2 + 3} \right] \right) + \left( \frac{1}{2} \left[ \frac{-2x_1 + \widetilde{\theta}_2 x_1 + 4}{x_2 + 1} \right] \right) + \left( \frac{1}{4} \left[ -x_1 - \widetilde{\theta}_3 x_1 + x_2 \right] \right) \right\}$$

Subject to

$$-x_1 + 4x_2 \le 0$$
$$2x_1 - x_2 \le 8$$
$$x_1 \le 4$$
$$x_1, x_2 \ge 0$$
$$0.05 \le \theta_1 \le 0.95$$
$$0.1 \le \theta_2 \le 1.15$$
$$0.06 \le \theta_3 \le 0.84$$

**Linearization technique**:
Let $\gamma_1 = x_1\theta_1, \gamma_2 = x_1\theta_2, \gamma_3 = x_1\theta_3$. The problem $p_2(w)$ can be written as

$$\max = \left\{ \left( \frac{1}{4} \left[ \frac{x_1 + 2\gamma_1 - 4}{-x_2 + 3} \right] \right) + \left( \frac{1}{2} \left[ \frac{-2x_1 - \gamma_2 + 4}{x_2 + 1} \right] \right) + \left( \frac{1}{4} \left[ -x_1 - \gamma_3 + x_2 \right] \right) \right\}$$

Subject to

$$-x_1 + 4x_2 \leq 0$$
$$2x_1 - x_2 \leq 8$$
$$x_1 \leq 4$$
$$x_1, x_2 \geq 0$$
$$0.05x_1 \leq \gamma_1 \leq 0.95x_1$$
$$0.1x_1 \leq \gamma_2 \leq 1.15x_1$$
$$0.06x_1 \leq \gamma_3 \leq 0.84x_1$$

By using the parametric approach of Dinkelbaeh [4] and Jagannathan [7], the above problem will take the following form:

$$p_3(w) : \max = \left\{ \begin{array}{l} \left(\frac{1}{4}\left[(x_1 + 2\gamma_1 - 4) - \lambda_1^*(-x_2 + 3)\right]\right) + \\ \left(\frac{1}{2}\left[(-2x_1 - \gamma_2 + 4) - \lambda_2^*(x_2 + 1)\right]\right) + \\ \left(\frac{1}{4}\left[-x_1 - \gamma_3 + x_2\right]\right) \end{array} \right\}$$

Subject to

$$-x_1 + 4x_2 \leq 0$$
$$2x_1 - x_2 \leq 8$$
$$x_1 \leq 4$$
$$x_1, x_2 \geq 0$$
$$0.05x_1 \leq \gamma_1 \leq 0.95x_1$$
$$0.1x_1 \leq \gamma_2 \leq 1.15x_1$$
$$0.06x_1 \leq \gamma_3 \leq 0.84x_1$$

Starting with $\lambda_1^* = z_1(0, 0, 0.05) = -\frac{4}{3}$, $\lambda_2^* = z_2(0, 0, 0.1) = 4$
Case 1: Let $x_1 = 0$ the problem can be written as:

$$p_4(w) : \max = \left\{\left(\frac{1}{4}\left[(-4) - \left(-\frac{4}{3}\right)(-x_2 + 3)\right]\right) + \left(\frac{1}{2}\left[(4) - (4)(x_2 + 1)\right]\right) + \left(\frac{1}{4}\left[x_2\right]\right)\right\}$$

Subject to

$$4x_2 \leq 0$$
$$x_2 \leq 8$$
$$x_2 \leq 0$$

$$p_4(w) : \max = \left\{\left(-\frac{1}{3}x_2\right) - (2x_2) + \left(\frac{1}{4}\left[x_2\right]\right)\right\}$$

Subject to

$$4x_2 \leq 0$$
$$x_2 \leq 8$$
$$x_2 \leq 0$$

By solving problem $p_4(w)$ the max $= 0$ at $(x_1^*, x_2^*) = (0, 0)$ substitute in $p_2(w)$ the optimal solution will be 1.666667
Case 2: Let $x_1 > 0$ the problem can be written as:

$$\begin{aligned} p_3(w) : \max = \Big\{ & \left(\frac{1}{4}\left[(x_1 + 2\gamma_1 - 4) - \left(-\frac{4}{3}\right)(-x_2 + 3)\right]\right) \\ & + \left(\frac{1}{2}\left[(-(2x_1 + \gamma_2) + 4) - (4)(x_2 + 1)\right]\right) + \left(\frac{1}{4}\left[-(x_1 + \gamma_3) + x_2\right]\right)\Big\} \end{aligned}$$

Subject to

$$\begin{aligned}
-x_1 + 4x_2 &\le 0\\
2x_1 - x_2 &\le 8\\
x_1 &\le 4\\
x_1 &\ge 1\\
x_1, x_2 &\ge 0\\
0.05x_1 \le \gamma_1 &\le 0.95x_1\\
0.1x_1 \le \gamma_2 &\le 1.15x_1\\
0.06x_1 \le \gamma_3 &\le 0.84x_1
\end{aligned}$$

$$p_5(w) : \max = \left\{ \left( \tfrac{1}{4} \left[ (x_1 + 2\gamma_1 - \tfrac{4}{3}x_2 \right] \right) + \left( \tfrac{1}{2} \left[ (-2x_1 - \gamma_2 - 4x_2 \right] \right) + \left( \tfrac{1}{4} \left[ -(x_1 + \gamma_3) + x_2 \right] \right) \right\}$$

Subject to

$$\begin{aligned}
-x_1 + 4x_2 &\le 0\\
2x_1 - x_2 &\le 8\\
x_1 &\le 4\\
x_1 &\ge 1\\
x_1, x_2 &\ge 0\\
0.05x_1 \le \gamma_1 &\le 0.95x_1\\
0.1x_1 \le \gamma_2 &\le 1.15x_1\\
0.06x_1 \le \gamma_3 &\le 0.84x_1
\end{aligned}$$

Solving problem $p_5(w)$, we obtain max $= -0.59$ at $(x_1^*, x_2^*) = (1, 0)$ and then substituting in $p_2(w)$ the $\alpha$-optimal solution will be 0.593333334 from the previous two cases, we notice that the $\alpha$-optimal solution of problem $p_2(w)$ is $(0, 0, 0.95, 0.1, 0.06)$ which gives the maximum value 1.666666667, on the other hand ,this is an $\alpha$-efficient solution for the (FMOINLFP) under consideration.

## 7. Conclusion

This paper has dealt with a fuzzified version of a multiobjective integer nonlinear fractional programming problem (FMOINLFP) in which fuzzy parameters are involved in the objective functions. In order to defuzzify this problem, the concept of $\alpha$-level set of the fuzzy number has been given. For obtaining an $\alpha$-efficient solution to the formulated problem (FMOINLPP), a linearization technique has been proposed to develop the solution process.

Though the computational experience is limited, our algorithm appears to be fairly efficient. Despite its simplicity, the proposed solution algorithm may be considered as evidence that a host of other fractional optimization problems can be effectively tackled by solving a sequence of $2^r$ feasibility problems.

In our opinion, the results of the illustrative example show that the nine digits is achievable in the solution steps since, these result using our proposed algorithm compared with direct LINGO software methodology will give the same solution in less than 30 iterations while more than 300 iterations are carried out using LINGO.

However, further points must be discussed in the area of (FMOINLPP) for different values of $\alpha$-level sets and the stability of the corresponding $\alpha$-efficient solutions should be investigated. It is recommended to suggest a solution algorithm to large-scale fuzzy multiobjective integer nonlinear fractional programming problems. It should therefore prove worthwhile to examine the convergence of the developed algorithm in this paper.

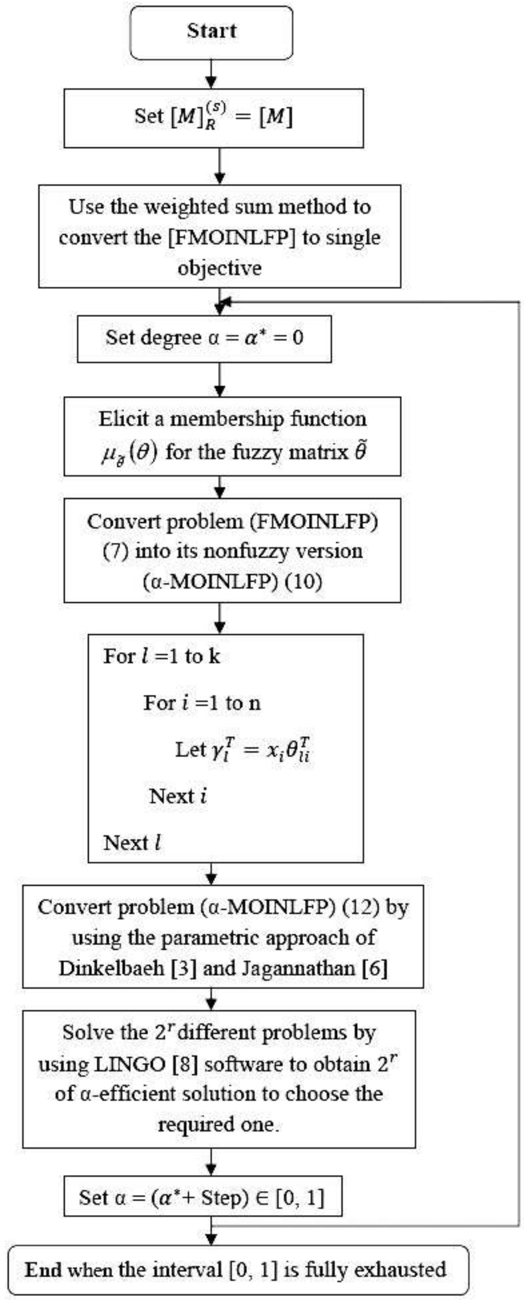


FIGURE 2. Flowchart for solution algorithm

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